

2007

ECONOMICS

THIRD PAPER

(Mathematical Methods for Economic Analysis)

Full Marks: 80

Time: 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions: $2 \times 4 = 8$

- a) Show that the determinant of a triangular matrix is equal to the products of the diagonal elements.
- b) Why is it necessary to add a constant term while integrating a function?
- c) A function $y = f(x_1, x_2)$ has to be maximized subject to two equality constraints

$$g_1(x_1, x_2) = c_1$$

$$g_2(x_1, x_2) = c_2$$

State the necessary and sufficient conditions of maximization.

(d) Distinguish between "mixed strategy and dominated strategy.

2. Answer any *four* of the following:

- a) Given the demand function

$$Q = \sqrt{60 - \frac{3}{2}P}$$

Where Q is quantity demand and P is price. Obtain consumer's surplus when

$p = 16$.

- b) "Imposition of specific sales tax not only changes the usual equilibrium condition of a monopolist but also results in a smaller output and higher price." Justify the above statement using optimization technique.

- c) A technology matrix is given as under:

	Steel	Coal
Steel	0.4	0.1
Coal	0.7	0.6
Labour	5	2

- Determine the equilibrium price if wage rate is Rs 10 per Labour Day.
- (d) Formulate a general transportation problem to linear programming where there are three factories for production and five destinations to supply the products.
- (e) In the competitive market model
- $$D=16-2P$$
- $$S = -4 +2(P- t)$$

where D, S, P and t denote demand, supply, price and sales tax rate. Obtain the rate of tax which will maximize total tax revenue of the government.

{f} Distinguish between static and dynamic input-output models with examples.

3. Answer any three of the following:

8×3=24

- a) Find out the general method of solution of a game where there are two players having m strategies each in the case of mixed strategies.
- b) Under joint production, a perfectly competitive firm sells two products q₁ and q₂ at prices Rs 10 and Rs 15 per unit of output respectively. The joint cost function of the firm is given by
- $$C=4q_1^2+2q_2^2+4q_1q_2$$
- Find profit maximizing output levels q₁ and q₂ and maximum profit.
- (c) Given the national income model

$$Y = C + I_0 + G_0$$

$$C = a + b(Y - T) \quad (a > 0, 1 > b > 0)$$

$$T = \delta Y \quad (0 < \delta < 1)$$

where Y, C, T, I₀ and G₀ denote income, consumption, income tax, investment and government expenditure respectively. Solve the above model using Cramer's rule.

- (d) Solve the following

$$\frac{dy}{dx} + 2x = 0$$

with initial condition y(0) = 5.

- (e) Let the demand function of a firm under monopolistic competition be given by

$$P = 118 - 3Q + 4\sqrt{A}$$

where P is price, Q is quantity and A is advertisement expenditure. If the total cost function is given by $C = 4Q^2 + 10Q + A$, find the value of A, Q and P that maximizes the profit of the firm.

4. Answer any two of the following:

12×2=24

- (a) (i) Find the optimal strategies for the participants in a two-person zero-sum game with the following profit matrix :

$$\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

What is the value of the game?

- (ii) What is Nash equilibrium? Explain with a suitable example.

(b) Given the demand and supply functions for cobweb model:

$$Q_{dt} = 10 - 2P_t$$

$$Q_{st} = -5 + 3P_{t-1}$$

Find the inter-temporal equilibrium price and also determine whether you will get stable equilibrium.

(c) A producer desires to minimize the cost of production

$$C = 16K + 4L$$

where K and L are capital and labour respectively subject to the production function given

$$Q = 5 K^{1/2} L^{1/2}$$

Find out the equilibrium combination of inputs (K and L) in order to minimize the cost of production when output $Q=40$.

(d) Maximize $f = 2X + 5Y$

subject to

$$X + 4Y \leq 24$$

$$3X + Y \leq 21$$

$$X + Y \leq 9$$

$$X, Y \geq 0$$