

2007

MATHEMATICS

FOURTH PAPER

(Tensors and Mechanics)

Full Marks: 80

Time: 3 hours

The figures in the margin indicate full marks for the questions

PART-A (Objective-type Questions)

(Marks: 32)

1. Answer the following questions:

1×10=10

- (a) What is the relation between contravariant and covariant vectors with reference to rectangular coordinate transformation?
- (b) Why g_{ij} in $ds^2 = g_{ij} dx^i dx^j$ is called the fundamental tensor?
- (c) State the difference of $g_{ij} A^j$ and $B_{ij} A^j$, if any.
- (d) When can we shift the point of application of a force F to any point on its line of action?
- (e) A system of forces acting on a rigid body is equivalent to _____. (Fill up completely)
- (f) If P and Q are two non-intersecting forces whose directions are perpendicular, the distances of the central axis from their lines of action are as x : y ; what are x and y ?
- (g) At the clamped end of a beam, $\frac{dy}{dx} = 0$
State the physical condition.
- (h) If A , B and C are the moments of inertia of a body about three mutually perpendicular axes, then what is the condition so that $C=A+B$ holds?
- (i) What is the moment of inertia of a right circular cone about a slant height?
- (j) Are the velocity components in cylindrical polar coordinates orthogonal? Give reasons.

2. Answer any five parts of the following:

2×5=10

- (a) From the derivation principle of the law

$$A'^i = \frac{\partial x'^i}{\partial x^\alpha} A^\alpha$$

Explain the reason of the use of the name 'contravariant'.

- (b) Partial derivative of a tensor of rank one is not a tensor. Give an example that the difference of two such derivatives is a tensor. Justify.
- (c) Deduce the condition that a system of forces acting on a body be equivalent to a single force.
- (d) Derive the expressions of invariant quantities for a system of forces.
- (e) A circular disc of radius r oscillates about an axis perpendicular to its plane at a distance h from the centre. Find the length of simple equivalent pendulum.
- (f) Write down the general equations of motion of a rigid body and explain their individual importance.
- (g) What do you mean by intrinsic equation of a path described by a particle? Why is it termed so?
- (h) In a central orbit, described by a particle, the velocity at any point is inversely proportional to the distance of the point from the centre of force. Find the law of force.

3. Answer any four parts of the following:

3×4=12

- (a) Show that every second-order tensor A_{ij} (or A^{ij}) can be expressed as the sum of a symmetric and an antisymmetric tensors.
- (b) If ϵ_{ijk} is the permutation tensor, show that $\epsilon^{123}\epsilon_{123} = \pm 3$ in three dimensions.
- (c) Find the moment of inertia of a circular disc of radius a and of mass M about a tangent.
- (d) Show that the motion of a particle in a conservative field of force is independent of the path.
- (e) Show that the Central Axis for a system of forces is unique.
- (f) Show that the resultant for a system of forces acting on a body is the same irrespective of referential base point.

PART-B (Subjective-type Questions)

(Marks: 48)

4. (a) Derive the x^k – covariant derivative of the second-order contravariant tensor A^{ij} with respect to the fundamental tensor g_{ij} . 8
- (b) Evaluate the non-vanishing Christoffel's second brackets for the metric 8

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

Or,

- (i) Show that a vector of constant magnitude is orthogonal to its intrinsic derivative. 4
- (ii) Deduce the expression of Laplacian of a vector \bar{A} 4

5. (a) Any number of wrenches of the same pitch p act along the generators of the same system of the hyperboloid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Show that they will reduce to a single resultant provided their central axis is parallel to a generator of the cone 8

$$\left(p + \frac{bc}{a}\right)x^2 + \left(p + \frac{ca}{b}\right)y^2 + \left(p - \frac{ab}{c}\right)z^2 = 0$$

(b) Show that a given system of forces can be replaced by two forces can be replaced by two forces equivalent to the given system in an infinite number of ways and the tetrahedron formed by the two forces is of constant volume. 8

Or,

A heavy rod AB rests on three horizontal supports A, B and C; C being midway between them and ABC horizontal. If the rod be slightly deflected, find its form. Hence show that for maximum sag,

$$x = a \quad \text{and} \quad x = a \cdot \frac{1 + \sqrt{33}}{16}$$

6. (a) A particle of mass m moves in a resisting medium under a central attraction mP . Show that the equation of the orbit is

$$\frac{d^2u}{d\theta^2} + u = \frac{p}{h^2u^2}, \quad \text{where} \quad h = h_0 e^{-\int \frac{R}{u} dt}$$

R is the resistance of the medium per unit of mass and h_0 is the initial moment of momentum about the centre of force. 8

Or,

What is the clear physical situation that admits inverse square law? Discuss the dynamical significance of Kepler's laws of planetary motion.

(b) A particle moves on a smooth sphere under no forces except the pressure of the surface. Show that the path of it is given by the equation $\cot\theta = \cot\beta\cos\phi$ where θ and ϕ are its angular coordinates and $\theta = \beta$ initially. 8

Or,

A circular area can turn freely about a horizontal axis which passes through a point O of its circumference and is perpendicular to its plane. If the motion commences when the diameter through O is vertically above O, show that when the diameter has turned through an angle θ , the components of strain at O along and perpendicular to the diameter are respectively $\frac{W}{3}(7\cos\theta - 4)$ and $\frac{W}{3}\sin\theta$

Where W is the weight of the circular area.