

2007

MATHEMATICS

FIFTH PAPER

**(Differential Equations, Numerical Analysis and Computer Programming)**

Full Marks: 80

Time: 3 hours

*The figures in the margin indicate full marks for the questions*

**PART-A**  
**(Objective-type Questions)**

(Marks: 32)

1. (a) Justify that  $y = x^m$  is a particular integral of

$$\frac{d^2y}{dx^2} + P \frac{dy}{dx} + Qy = 0$$

If  $m(m-1) + Pm + Qx^2 = 0$

- (b) Give the geometrical interpretation of

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

- (c) Explain the role of indicial equation in finding the power series solution of a second-order ordinary differential equation.  
(d) When  $x = x_0$  is called an ordinary point, singular point, regular singular point and irregular singular point of the differential equation

$$\frac{d^2y}{dx^2} + P \frac{dy}{dx} + Qy = 0?$$

- (e) Expand the Wronskian  $W [y_1, y_2 \text{ and } y_3] (x)$  if  $y_1, y_2$ , and  $y_3$  are three linearly independent solutions.  
(f) State how the linear dependency or independency of solutions for a differential equation depends on the Wronskian.  
(g) State when the differential equation

$$\frac{dy}{dx} = f(x, y), y(x_0) = y_0$$

has unique solution.

(h) If  $y_1$  is a particular solution of the Riccati equation, then mention the way of finding the general solution. 2×8=16

2. (a) Under what situation Newton's divided difference formula is adopted?  
(b) Explain the difference between the Newton's interpolation and Lagrange's interpolation formula.  
(c) Write Simpson's  $\frac{1}{3}$ <sup>rd</sup> rule and trapezoidal rule. Explain the efficiency of one over the other.  
(d) Explain the role of the two '\*' in READ (\*,\*) statement in FORTRAN language.  
(e) Correct the program segment:

```
DO 10 I = 1, 10
  I = 5 + I
10 CONTINUE
```

- (f) Write  $\log_{10}(x)$  and  $\log_e(x)$  in FORTRAN.  
(g) Write FORTRAN statement and logical if statement in FORTRAN. 2×8=16  
(h) Write arithmetic if statement and logical if statement in FORTRAN. 2×8=16

### PART-B (Subjective-type Questions)

#### Section-I

#### (Differential Equations)

(Marks: 24)

**Answer any two questions**

3. Answer any three: 4×3=12

(a) Show that  $y=x$  is a part of the CF of

$$\frac{d^2 y}{dx^2} - x^2 \frac{dy}{dx} + xy = x$$

and hence solve it.

(b) Solve

$$\frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + (x^2 + 2)y = e^{\frac{1}{2}(x^2+2x)}$$

(c) Solve

$$\frac{d^2 y}{dx^2} - \cot x \frac{dy}{dx} - \sin^2 x \cdot y = \cos x - \cos^3 x$$

by removing the first derivative.

(d) Apply the method of variation of parameters to solve

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$$

4. Answer part (a) and any one of (b), (c):

(a) Show that  $x=0$  is an ordinary point for the differential equation

$$\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} - y = 0$$

And hence solve it in powers of  $x$ . 7

(b) Solve the simultaneous equations 5

$$\frac{dx}{dt} - 7x + y = 0; \frac{dy}{dt} - 2x - 5y = 0$$

(c) Show that the condition of integrability is satisfied by the equation

$$Z(z-y)dx + (z+x)zdy + x(x+y)dz = 0 \text{ and solve it.} \quad 5$$

5. (a) Establish the following result : 7

The general solution of the linear partial differential equation  $P_p + Q_q = R$  is  $f(u,v)=0$  where  $f$  is an arbitrary function and  $u(x, y, z) = c_1$  and  $v(x, y, z) = c_2$  form a solution of the equations

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

(b) Use Charpit's method to find a complete integral of the equation

$$2zx - px^2 - 2qxy + pq = 0 \quad 5$$

**Section-II**  
**(Numerical Analysis and Computer Programming)**

(Marks: 24)

Answer any two questions

6. (a) The population of a country in census are as under. Estimate the population for the year 1925: 6

Year (x)	:	1891	1901	1911	1921	1931
Population (y) (in thousand)	:	46	66	81	93	101

(b) Derive the Newton's divided difference interpolation formula. Mention when this formula is used. 5+1=6

7. (a) Show that the rate of convergence of the Newton-Raphson method is quadratic. 6

(b) Use the following data to find  $f'(5)$ : 6

x :	0	2	3	4	7	9
f(x) :	4	26	58	112	466	922

8. (a) Write a program in FORTRAN to find the largest and the smallest of 50 gives numbers. 6

6

(b) Write a program in FORTRAN to solve the equation

$$x^5 + 3\cos(x) - 2 \tan(x) = 0$$

Using the algorithm of Newton-Raphson method. 6